Class- X Session- 2022-23

Subject- Mathematics (Basic)

Sample Question Paper - 2

with Solution

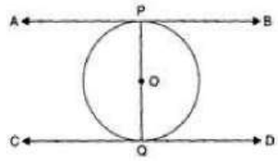
Time Allowed: 3 Hrs. **Maximum Marks : 80**

General Instructions:

1. This Question Paper has 5 Sections A-E.
2. Section **A** has 20 MCQs carrying 1 mark each
3. Section **B** has 5 questions carrying 02 marks each.
4. Section **C** has 6 questions carrying 03 marks each.
5. Section **D** has 4 questions carrying 05 marks each.
6. Section **E** has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. All Questions are compulsory. However, an internal choice in 2 Qs of 5 marks, 2 Qs of 3 marks and 2 Questions of 2 marks has been provided. An internal choice has been provided in the 2marks questions of Section E
8. Draw neat figures wherever required. Take π =22/7 wherever required if not stated.

Section A

1. The distance between two parallel tangents of a circle of radius 3 cm is **[1]**



a) 6 cm b) 3 cm

c) 4.5 cm d) 12 cm

2. The abscissa of any point on the y-axis is **[1]**

a) 0 b) 1

c) y d) -1

3. In the fourth quadrant, **[1]**

a) x is +ve, y is -ve b) x is -ve, y is -ve

c) x is +ve, y is +ve d) x is -ve, y is +ve

4. If the probability of winning a game is 0.4 then the probability of losing it, is **[1]**

a) None of these b) 0.6

c) 0.4 d) 0.96

5. If the line segment joining the points A (x_1, y_1) and B(x_2, y_2) is divided by a point P in the ratio 1 : k internally, then the coordinates of the point P are **[1]**

a) $\left(\frac{x_2-kx_1}{1+k}, \frac{y_2-ky_1}{1+k}\right)$

b) $\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

c) $\left(\frac{x_2+kx_1}{1-k}, \frac{y_2+ky_1}{1-k}\right)$

d) $\left(\frac{x_1+kx_2}{1+k}, \frac{y_1+ky_2}{1+k}\right)$

6. The graph of y + 2 = 0 is a line **[1]**

a) making an intercept of -2 on the y-axis

b) parallel to the y-axis at a distance of 2 units to the left of y-axis

c) making an intercept of -2 on the x-axis

d) parallel to the x-axis at a distance of 2 units below the x-axis

7. A card is selected from a deck of 52 cards. The probability of its being a red face card is **[1]**

a) $\frac{3}{13}$

b) $\frac{1}{2}$

c) $\frac{2}{12}$

d) $\frac{3}{26}$

8. Which of the following cannot be the probability of a event? **[1]**

a) $\frac{17}{16}$

b) $\frac{1}{3}$

c) 0.1

d) 3%

9. The ratio of the total surface area to the lateral surface area of a cylinder with base radius 80 cm and height 20 cm is **[1]**

a) 5 : 1

b) 4 : 1

c) 2 : 1

d) 3 : 1

10. If the sum of a number and its reciprocal is $2\frac{1}{2}$, then the numbers are **[1]**

a) 3 and $\frac{1}{3}$

b) 1 and $\frac{3}{2}$

c) None of these

d) 2 and $\frac{1}{2}$

11. In a right triangle ABC, $\angle B = 90^\circ$ and 2 AB = $\sqrt{3}$AC, then $\angle C$ is **[1]**

a) 90°

b) 60°

c) 75°

d) 30°

12. The total number of factors of a prime number is: **[1]**

a) 2

b) 1

c) 3

d) 0

13. If the coordinates of a point are (-5, 11), then its abscissa is **[1]**

a) -5

b) 11

c) 5

d) -11

14. $2x^2 - 3x + 2 = 0$ have **[1]**

a) Real and Distinct roots

b) Real and Equal roots

c) Real roots

d) No Real roots

15. The mean of 20 numbers is zero. Of them, at the most, how many may be greater than zero? **[1]**

a) 1

b) 0

c) 10

d) 19

16. In a right $\triangle ABC$, AC is the hypotenuse of length 10cm. If $\angle A = 30°$, then the area of the triangle is **[1]**

a) $25\sqrt{3}cm^2$

b) $25cm^2$

c) $\frac{25}{3}\sqrt{3}cm^2$

d) $\frac{25}{2}\sqrt{3}cm^2$

17. The pair of equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ have **[1]**

a) a unique solution

b) infinitely many solutions

c) no solution

d) exactly two solutions

18. $(1 + \sqrt{2}) + (1 - \sqrt{2})$ is **[1]**

a) a rational number

b) a non-terminating decimal

c) None of these

d) an irrational number

19. **Assertion (A):** If ratio of perimeters of two similar triangles is 6 : 11, then ratio of their corresponding medians is also 6 : 11. **[1]**
Reason (R): Converse of B.P.T. states that if two sides of a triangle are divided by a line in equal ratio then the line is parallel to the third side.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. **Assertion (A):** H.C.F. of smallest prime and smallest composite is 2. **[1]**
Reason (R): Smallest prime is 2 and smallest composite is 4 so their H.C.F. is 2.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

Section B

21. On comparing the ratios $\frac{a_1}{a_2}, \frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the pair of linear equations intersect at a point, are parallel or coincide: $9x + 3y + 12 = 0$; $18x + 6y + 24 = 0$ **[2]**

OR

The ratio of incomes of two persons is 9:7 and the ratio of their expenditures is 4:3. If each of them saves Rs.200 per month, find their monthly incomes.

22. A lot consists of 144 ball pens of which 20 are defective and the others are good. Nuri will buy a pen if it is good, but will not buy if it is defective. The shopkeeper draws one pen at random and gives it to her. What is the probability that:(i) she will buy it?(ii) she will not buy it? **[2]**

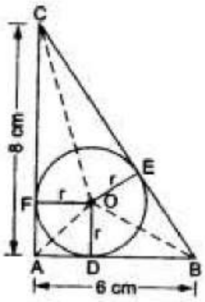
23. Find the point on the x-axis which is equidistant from (2, -5) and (-2, 9). **[2]**

24. Find the zeroes of a quadratic polynomial given as: $4s^2 - 4s + 1$ and also verify the relationship between the zeroes and the coefficients. **[2]**

25. ABC is a right triangle, right-angled at B, such that BC = 6 cm and AB = 8 cm, find the radius of circle. **[2]**

OR

In the given figure, ABC is a right-angled triangle with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of the radius of the inscribed circle.



Section C

26. Solve the pair of linear equations: **[3]**
$152x - 378y = -74$; $-378x + 152y = -604$.

27. If $\angle B$ and $\angle Q$ are acute such that $\sin B = \sin Q$, then prove that $\angle B = \angle Q$. **[3]**

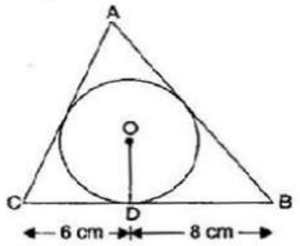
28. Show that $5 - \sqrt{3}$ is irrational. **[3]**

OR

If the HCF of 657 and 963 is expressible in the form of $657x + 963 \times (-15)$, find the value of x.

29. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of **[3]**

lengths 8 cm and 6 cm respectively (see figure). Find the sides AB and AC.



OR

AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.

30. Diagonals of a trapezium PQRS intersect each other at the point O, PQ $\parallel$ RS and PQ = 3 RS. Find the ratio of the areas of triangles $\triangle$POQ and $\triangle$ROS. **[3]**

31. The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle θ with the ground level such that $\tan\theta = 15/8$ then find the height of the kite from the ground. Assume that there is no slack in the string. **[3]**

Section D

32. In the figure, OB is the perpendicular bisector of the line segment DE, FA $\perp$ OB and F E intersect OB at point C. Prove that $\frac{1}{\mathrm{OA}} + \frac{1}{\mathrm{OB}} = \frac{2}{\mathrm{OC}}$. **[5]**

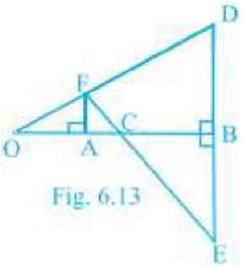


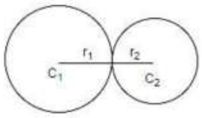
Fig. 6.13

33. If the equation $\left(1+m^2\right)x^2 + 2mcx + \left(c^2 - a^2\right) = 0$ has equal roots, prove that $c^2 = a^2\left(1+m^2\right)$ **[5]**

OR

If the factory kept increasing its output by the same percentage every year. Find the percentage, if it is known that the output doubles in the last two years.

34. Two farmers have circular plots. The plots are watered with the same water source placed in the point common to both the plots as shown in the figure. The sum of their areas is 130π and the distance between their centres is 14 m.Find the radii of the circles.What value is depicted by the farmers? **[5]**



OR

Find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres (Use π = 22/7).

35. The monthly income of 100 families are given as below: **[5]**

Income in (in ₹.)	Number of families
0-5000	8
5000-10000	26
10000-15000	41
15000-20000	16
20000-25000	3
25000-30000	3
30000-35000	2
35000-40000	1

Calculate the modal income.

Section E

36. **Read the text carefully and answer the questions:** **[4]**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

(i) How much distance did she cover in pacing 6 flags on either side of center point?

(ii) Represent above information in Arithmetic progression

(iii) How much distance did she cover in completing this job and returning to collect her books?

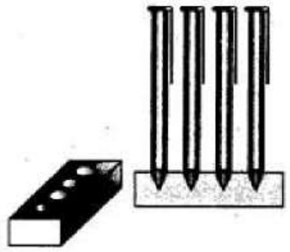
OR

What is the maximum distance she travelled carrying a flag?

37. **Read the text carefully and answer the questions:** **[4]**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:
A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The

radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.



(i) The volume of the cuboidal part.

(ii) The volume of wood in the entire stand.

(iii) Total volume of conical depression.

OR

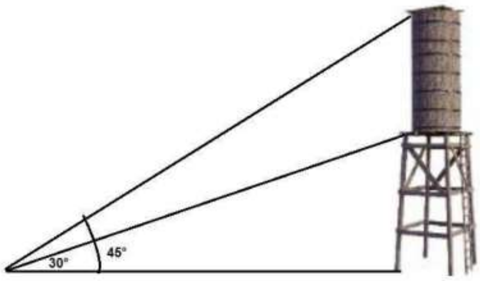
If the cost of wood used is ₹10 per cm^3, then the total cost of making the pen stand.

38. **Read the text carefully and answer the questions:** **[4]**

In a society, there are many multistory buildings. The RWA of the society wants to install a tower and a water tank so that all the households can get water without using water pumps.

For this they have measured the height of the tallest building in the society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.

From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 300. the angle of elevation of the top of the water tank is 45^o.



(i) What is the height of the tower?

(ii) What is the height of the water tank?

(iii) At what distance from the bottom of the tower the angle of elevation of the top of the tower is 45°.

OR

What will be the angle of elevation of the top of the water tank from the place at $\frac{40}{\sqrt{3}}$ m from the bottom of the tower.

Solution

Section A

1. **(a)** 6 cm

   Explanation: Since the distance between two parallel tangents of a circle is equal to the diameter of the circle.
   Given: Radius (OP) = 3 cm
   $\therefore$ Diameter = 2 × Radius = 2 × 3 = 6 cm
2. **(a)** 0

   Explanation: Since coordinates of any point on y-axis is (0, y)
   Therefore, the abscissa is 0.
3. **(a)** x is +ve, y is -ve

   Explanation: In the fourth quadrant, x is positive, y is negative.
   i.e the value of x is called abscissa which is positive and the value of y is called coordinate which is negative in the 4th quadrant
4. **(b)** 0.6

   Explanation: P(losing the game) = 1 - P (winning the game) = (1 - 0.4) = 0.6
5. **(b)** $\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$

   Explanation: Let coordinates of P be (x, y) which divides the line joining A(x_1, y_1) and B(x_2, y_2) in the ratio 1 : k
   $m_1 : m_2 = 1 : k$
   $\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$
   $= \frac{1\times x_2+k\times x_1}{1+k} = \frac{x_2+kx_1}{1+k}$
   And $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$
   $= \frac{1\times y_2+k\times y_1}{1+k}$
   $= \frac{y_2+ky_1}{1+k}$
   $\therefore P\left(\frac{x_2+kx_1}{1+k}, \frac{y_2+ky_1}{1+k}\right)$
6. **(d)** parallel to the x-axis at a distance of 2 units below the x-axis

   Explanation: The graph of $y + 2 = 0$ is a line parallel to the x-axis at a distance of 2 units below the x-axis.
7. **(d)** $\frac{3}{26}$

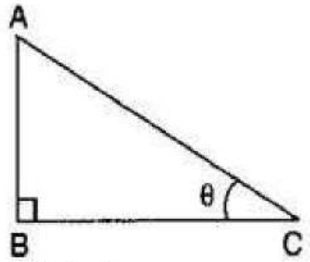
   Explanation: In a deck of 52 cards, there are 12 face cards i.e. 6 red (3 hearts and 3 diamonds) and 6 black cards (3 spade and 3 clubs)
   So, probability of getting a red face card = 6/52 = 3/26
8. **(a)** $\frac{17}{16}$

   Explanation: Since, probability of an event always lies between 0 and 1.
   Probability of any event cannot be more than 1 or negative as $\frac{17}{16} > 1$
9. **(a)** 5 : 1

   Explanation: Ratio of the total surface area to the lateral surface area $= \frac{\text{Total surface area}}{\text{Lateral surfice area}}$
   $= \frac{2\pi r(h+r)}{2\pi rh}$
   $= \frac{h+r}{h}$
   $= \frac{(20+80)}{20}$

$= \frac{100}{20}$
$= \frac{5}{1}$
$= 5 : 1$
Hence, the required ratio is 5:1

10. **(d)** 2 and $\frac{1}{2}$
Explanation: Let the one number be x then its reciprocal will be $\frac{1}{x}$ According to question,
$x + \frac{1}{x} = 2\frac{1}{2}$
$\Rightarrow \frac{x^2+1}{x} = \frac{5}{2}$
$\Rightarrow 2x^2 + 2 = 5x$
$\Rightarrow 2x^2 - 5x + 2 = 0$
Using factorisation method
$\Rightarrow 2x^2 - 4x - x + 2 = 0$
$\Rightarrow 2x(x-2) - 1(x-2) = 0$
$\Rightarrow (x-2)(2x-1) = 0$
$\Rightarrow x - 2 = 0$ and $2x - 1 = 0$
$\Rightarrow x = 2$ and $x = \frac{1}{2}$
Therefore, the numbers are 2 and $\frac{1}{2}$.

11. **(b)** 60°
Explanation:


Given: $2AB = \sqrt{3}AC$
Let DC be θ
$\Rightarrow AB = \frac{\sqrt{3}}{2}AC$
$\therefore \sin\theta = \frac{AB}{AC}$
$\Rightarrow \sin\theta = \frac{\frac{\sqrt{3}}{2}AC}{AC}$
$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$
$\Rightarrow \sin\theta = \sin 60^\circ \Rightarrow \theta = 60^\circ$

12. **(a)** 2
Explanation: The total number of factors of a prime number = 2 i.e. 1 and itself

13. **(a)** -5
Explanation: Since x-coordinate of a point is called abscissa.
Therefore, the abscissa is -5.

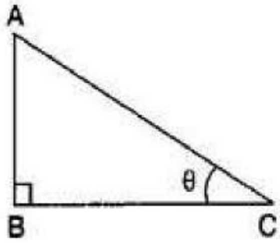
14. **(d)** No Real roots
Explanation: $D = b^2 - 4ac$
$D = (-3)^2 - 4 \times 2 \times 2$
$D = 9 - 16$
$D = -7$
$D < 0$. Hence No Real roots.

15. **(d)** 19
Explanation: Mean of 20 numbers = 0
Hence, sum of 20 numbers = $0 \times 20 = 0$

Now, the mean can be zero if
sum of 10 numbers is (S) and the sum of remaining 10 numbers is (-S),
sum of 11 numbers is (S) and the sum of remaining 9 numbers is (-S), ...
sum of 19 numbers is (S) and the 20^{th} number is (-S), then their sum is zero.
So, at the most, 19 numbers can be greater than zero.

16. **(d)** $\frac{25}{2}\sqrt{3}cm^2$

Explanation:



In triangle ABC, AC is hypotenuse of the length = 10 cm$\angle A = 30^\circ$ Now, $\sin 30^\circ = \frac{BC}{AC}$

$\Rightarrow \frac{1}{2} = \frac{BC}{10}$

$\Rightarrow BC = \frac{10}{2} = 5 \text{ cm}$

Now, AB = $\sqrt{(AC)^2 - (BC)^2}$

$= \sqrt{(10)^2 - (5)^2}$

$= \sqrt{100 - 25} = \sqrt{75} = 5\sqrt{3} \text{ cm}$

$\therefore \text{ar}(\Delta ABC) = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 5\sqrt{3} \times 5 = \frac{25\sqrt{3}}{2}$ sq. cm

17. **(c)** no solution
Explanation: Given, equations are
x + 2y + 5 = 0, and
- 3x - 6y + 1 = 0.
Comparing the equations with general form:
$a_1x + b_1y + c_1 = 0$
$a_2x + b_2y + c_2 = 0$
Here, $a_1 = 1, b_1 = 2, c_1 = 5$
And $a_2 = -3, b_2 = -6, c_2 = 1$
Taking the ratio of coefficients to compare

$\frac{a_1}{a_2} = \frac{-1}{3}, \frac{b_1}{b_2} = \frac{-1}{3}, \frac{c_1}{c_2} = \frac{5}{1}$

So $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$
This represents a pair of parallel lines.
Hence, the pair of equations has no solution.

18. **(a)** a rational number
Explanation: $(1 + \sqrt{2}) + (1 - \sqrt{2}) = 1 + \sqrt{2} + 1 - \sqrt{2} = 1 + 1 = 2$ And 2 is a rational number.
Therefore the given number is rational number.

19. **(c)** A is true but R is false.
Explanation: A is true but R is false.

20. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: Smallest prime is 2 and smallest composite is 4 so H.C.F. of 2 and 4 is 4.

Section B

21. Given equations are

$9x + 3y + 12 = 0$
$18x + 6y + 24 = 0$
Comparing equation $9x + 3y + 12 = 0$ with $a_1x + b_1y + c_1 = 0$
and $18x + 6y + 24 = 0$ with
$a_2x + b_2y + c_2 = 0$,
We get, $a_1 = 9, b_1 = 3, c_1 = 12, a_2 = 18, b_2 = 6, c_2 = 24$
We have $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ because $\frac{9}{18} = \frac{3}{6} = \frac{12}{24} \Rightarrow \frac{1}{2} = \frac{1}{2} = \frac{1}{2}$
Hence, lines are coincident.

OR

Let the common ratio term of income be x and expenditure be y.
So, the income of first person is Rs.9x and the income of second person is Rs.7x.
And the expenditures of first and second person is 4y and 3y respectively.
Then, Saving of first person =9x - 4y
and saving of second person = 7x - 3y
As per given condition
$9x - 4y = 200$
$\Rightarrow 9x - 4y - 200=0$ ... (i)
and, $7x - 3y = 200$
$\Rightarrow 7x - 3y - 200 =0$ ..... (ii)
Solving equation (i) and (ii) by cross-multiplication, we have
$\frac{x}{800-600} = \frac{-y}{-1800+1400} = \frac{1}{-27+28}$
$\frac{x}{200} = \frac{-y}{-400} = \frac{1}{1}$
$\Rightarrow x =200$ and $y =400$
So, the solution of equations is x = 200 and y = 400.
Thus, monthly income of first person = Rs.9x = Rs.(9 × 200)= Rs.1800
and, monthly income of second person = Rs.7x = Rs.(7× 200)= Rs. 1400

22. Total number of favourable outcomes = 144
$Proabibilty\ of\ the\ event = \frac{Number\ of\ favourble\ outcomes}{Total\ number\ of\ possible\ outcomes}$
   i. Number of non-defective pens = 144 - 20 = 124
   Number of favourable outcomes = 124
   Hence P (she will buy) = P (a non-defective pen) = $\frac{124}{144} = \frac{31}{36}$
   ii. Number of favourable outcomes = 20
   Hence P (she will not buy) = P (a defective pen) = $\frac{20}{144} = \frac{5}{36}$

23. We know that a point on the x-axis is of the form (x, 0). So, let the point P(x, 0) be equidistant from A(2, –5) and B(–2, 9). Then
PA = PB
$\Rightarrow PA^2 = PB^2$
$\Rightarrow (2 - x)^2 + (-5 - 0)^2 = (-2 - x)^2 + (9 - 0)^2$
$\Rightarrow 4 + x^2 - 4x + 25 = 4 + x^2 + 4x + 81$
$\Rightarrow - 4x + 25 = 4x + 81$
$\Rightarrow 8x = -56$
$\Rightarrow x = \frac{-56}{8} = -7$
Hence, the required point is (-7, 0)
Check:
$PA = \sqrt{\{2-(-7)\}^2 + (-5-0)^2}$

$= \sqrt{81+25} = \sqrt{106}$

$PB = \sqrt{\{-2-(-7)\}^2 + (9-0)^2}$

$= \sqrt{25-81} = \sqrt{106}$

$\because$ PA = PB

$\therefore$ Our solution is checked.

24. The given quadratic equation is $4s^2 - 4s + 1$

$= (2s)^2 - 2(2s)1 + 1^2$

As, we know $(a - b)^2 = a^2 - 2ab + b^2$, the above equation can be written as

$= (2s - 1)^2$

The value of $4s^2 - 4s + 1$ is zero when $2s - 1 = 0$, when, s = $\frac{1}{2}, \frac{1}{2}$

Therefore, the zeroes of $4s^2 - 4s + 1$ are $\frac{1}{2}$ and $\frac{1}{2}$

Sum of zeroes = $\frac{1}{2} + \frac{1}{2} = 1 = \frac{-(-4)}{4} = \frac{-(\text{coefficient of } s)}{\text{coefficient of } s^2}$

Product of zeroes = $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{\text{constant term}}{\text{coefficient of } s^2}$

Hence Verified.

25. Let ABC be the right angled triangle such that $\angle B = 90°$, BC = 6 cm, AB = 8 cm. Let O be the centre and r be the radius of the in circle.

AB, BC and CA are tangents to the circle at P, Nand M.

$\therefore$ OP = ON = OM = r (radius of the circle)

Area of the $\triangle ABC = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

By Pythagoras theorem,

$CA^2 = AB^2 + BC^2$

$\Rightarrow CA^2 = 8^2 + 6^2$

$\Rightarrow CA^2 = 100$

$\Rightarrow$ CA = 10 cm

Area of the $\triangle ABC$ = Area $\triangle OAB$ + Area $\triangle OBC$ + Area $\triangle OCA$

$24 = \frac{1}{2}r \times AB + \frac{1}{2}r \times BC + \frac{1}{2}r \times CA$

$24 = \frac{1}{2}r(AB + BC + CA)$

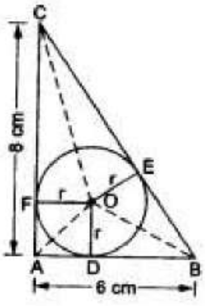
$\Rightarrow r = \frac{2 \times 24}{(AB+BC+CA)}$

$\Rightarrow r = \frac{48}{8+6+10}$

$\Rightarrow r = \frac{48}{24}$

$\Rightarrow$ r = 2 cm

OR



Join OA, OB and OC.

Draw $OD \perp AB, OE \perp BC$

and $OF \perp CA$

Then, OD = OE = OF = r cm.

$\therefore \quad \operatorname{ar}(\Delta ABC) = \frac{1}{2} \times AB \times AC$

$= \left(\frac{1}{2} \times 6 \times 8\right) \mathrm{cm}^2 = 24 \mathrm{cm}^2$

Now, $\operatorname{ar}(\Delta ABC) = \frac{1}{2} \times (\text{ perimeter of } \Delta ABC) \times r$

$\Rightarrow \quad 24 = \frac{1}{2} \times (AB + BC + CA) \times r$

$\Rightarrow \quad 24 = \frac{1}{2} \times (6 + 10 + 8) \times r$

$\Rightarrow r = 2 \ \left[\because BC^2 = AB^2 + AC^2 \Rightarrow BC = \sqrt{6^2 + 8^2} = 10\right]$

Hence, the radius of the inscribed circle is 2cm.

Section C

26. The given pair of linear equations is

152x - 378y = -74 ...(1)

-378x + 152y = -604 ....(2)

Adding equation (1) and equation (2), we get

-226x - 226y = -678

⇒ x + y = 3 ....(3) .....Dividing throughout by -226

Subtracting equation (2) from equation (1), we get 530x - 530y = 530

⇒ x - y = 1 .....(4) ....Dividing throughout by 530

Adding equation (3) and equation (4), we get 2x = 4

$\Rightarrow \ x = \frac{4}{2} = 2$

Subtracting equation (4) from equation (3), we get 2y = 2

$\Rightarrow \ y = \frac{2}{2} = 1$

Hence, the solution of the given pair of linear equations is x = 2, y = 1.

Verification: Substituting x = 2, y = 1,

We find that both the equations (1) and (2) are satisfied as shown below:

152x - 378y = (152)(2) - (378)(1)

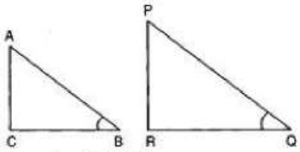
= 304 - 378 = -74

-378x + 152y = (-378)(2) + (152)(1)

= -756 + 152 = -604

27. Consider two right triangles ABC and PQR in which $\angle B$ and $\angle Q$ are the right angles.

We have,



In $\triangle ABC$

$\sin B = \frac{AC}{AB}$

and, In $\triangle PQR$

$\sin Q = \frac{PR}{PQ}$

$\because \quad \sin B = \sin Q$

$\Rightarrow \quad \frac{AC}{AB} = \frac{PR}{PQ}$

$\Rightarrow \quad \frac{AC}{PR} = \frac{AB}{PQ} = k \text{ (say) ...... (i)}$

⇒ AC = kPR and AB = kPQ .....(ii)

Using Pythagoras theorem in triangles ABC and PQR, we obtain

$AB^2 = AC^2 + BC^2$ and $PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2}$ and $QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \frac{BC}{QR} = \frac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \frac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}}$ [using (ii)]

$\Rightarrow \quad \frac{BC}{QR} = \frac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k$ ...(iii)

From (i) and (iii), we get

$\frac{AC}{PR} = \frac{AB}{PQ} = \frac{BC}{QR}$

$\Rightarrow \quad \Delta ACB - \Delta PRQ$ [By S.A.S similarity]

$\therefore \quad \angle B = \angle Q$

Hence proved.

28. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime numbers a and b (b $\neq$ 0) such that $5 - \sqrt{3} = \frac{a}{b}$

Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b-a}{b}$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

OR

Using Euclid's Division Lemma,

963 = 657 × 1 + 306

657 = 306 × 2 + 45

306 = 45 × 6 + 36

45 = 36 × 1 + 9

36 = 9 × 4 +0

$\therefore HCF(963, 657) = 9$

$Now\ it\ is\ given\ that$

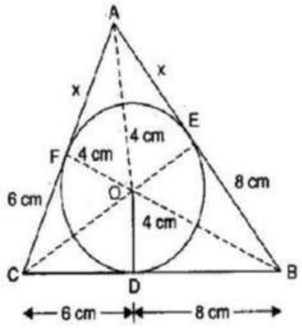
$HCF = 657x + 963(-15)$

$or\ 9 = 657x + 963(-15)$

$657x = 9 + 14445 = 14454$

$x = \frac{14454}{657} = 22$

29. Join OE and OF. Also join OA, OB and OC.



Since BD = 8 cm

$\therefore$ BE = 8 cm

[Tangents from an external point to a circle are equal]

Since CD = 6 cm

$\therefore$ CF = 6 cm

[Tangents from an external point to a circle are equal]

Let AE = AF = x

Since OD = OE = OF = 4 cm [Radii of a circle are equal]

$\therefore$ Semi-perimeter of $\triangle$ ABC = $\frac{(x+6)(x+8)+(6+8)}{2} = \frac{(2x+28)}{2}$ = (x + 14)cm

$\therefore$ Area of $\triangle$ABC = $\sqrt{s(s-a)(s-b)\ (s-c)}$

$= \sqrt{(x+14)(x+14-14)(x+14-\overline{x+8})(x+14-\overline{x+6})}$

$= \sqrt{(x+14)(x)(8)(6)}\,\text{cm}^2$

Now, Area of ΔABC = Area of $\triangle$OBC + Area of $\triangle$OCA + Area of $\triangle$OAB

$\Rightarrow \sqrt{(x+14)(x)(8)(6)} = \frac{(6+8)4}{2} + \frac{(x+6)4}{2} + \frac{(x+8)4}{2}$

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ = 28 + 2x + 12 + 2x + 16

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ = 4x + 56

$\Rightarrow \sqrt{(x+14)(x)(8)(6)}$ =4(x + 14)

Squaring both sides,

(x + 14) (x) (6) (8) = 16$(x + 14)^2$

$\Rightarrow$ 3x = x + 14

$\Rightarrow$ 2x = 14

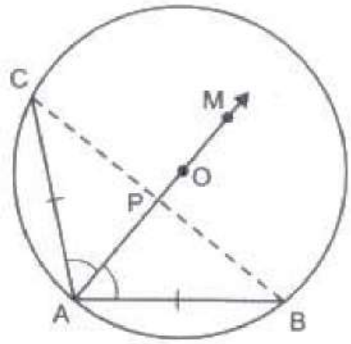
$\Rightarrow$ x = 7

$\therefore$ AB = x + 8 = 7 + 8 = 15 cm

And AC = x + 6= 7 + 6 = 13 cm

OR

Given: AB =AC and AM is the bisector of $\angle BAC$.



To prove: AM passes through O.

Construction: Join BC. Let AM intersect BC at P.

Proof: In ΔBAP and ΔCAP

AB = AC [Given]

$\angle BAP = \angle CAP$ [Given]

And AP = BP [Common side]

$\therefore \Delta BAP \cong \Delta CAP$ [By SAS congruency]

$\therefore \angle BPA = \angle CPA$ [By C.P.C.T.]

And CP = PB

But $\angle BPA + \angle CPA = 180^\circ$ [Linear pair $\angle s$]

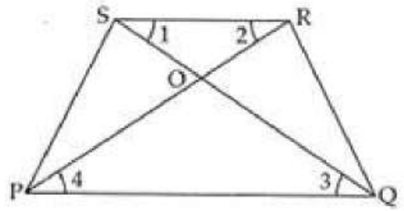
$\therefore \angle BPA = \angle CPA = 90^\circ$

$\therefore$ AP is perpendicular bisector of the chord BC, which will pass through the centre O on being produced.

Hence, AM passes through O.

30. Diagonals of a trapezium PQRS intersect each other at the point O, PQ || RS and PQ = 3 RS. We have to find the ratio of the areas of triangles $\triangle$POQ and $\triangle$ROS.

Given: PQRS is a trapezium with PQ || RS and PQ = 3RS



To find: $\frac{\text{ar}(\Delta POQ)}{\text{ar}(\Delta ROS)}$

In $\triangle$POQ and $\triangle$ROS,

PQ $\parallel$ RS [Given]

$\therefore \angle 3 = \angle 1$ [Alt. int. $\angle$s]

$\angle 4 = \angle 2$ [Alt. int. $\angle$s]

$\therefore \triangle POQ \sim \triangle ROS$ [By AA similarity criterion]

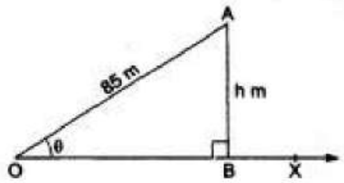
So, $\frac{\text{ar}(\Delta POQ)}{\text{ar}(\Delta ROS)} = \left(\frac{PQ}{RS}\right)^2$ [By area theorem]

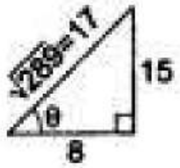
But, PQ = 3RS [Given]

$\Rightarrow \frac{\text{ar}(\Delta POQ)}{\text{ar}(\Delta ROS)} = \left(\frac{3RS}{RS}\right)^2 = \frac{9}{1}$

Hence, the required ratio is 9 : 1.

31. Let OX be the horizontal ground and let A be the position of the kite. Let O be the position of the observer and OA be the string. Draw $AB \perp OX$.





Then, $\angle BOA = \theta$ such that $\tan\theta = \frac{15}{8}$, $OA = 85m$ and $\angle OBA = 90^\circ$.

Let AB = h m.

From right ΔOBA, we have

$\frac{AB}{OA} = \sin\theta = \frac{15}{17}$ $\left[\because \tan\theta = \frac{15}{8} \Rightarrow \sin\theta = \frac{15}{17}\right]$

$\Rightarrow \frac{h}{85} = \frac{15}{17} \Rightarrow h = \frac{15}{17} \times 85 = 75.$

Section D

32. In $\triangle$AOF and $\triangle$BOD

$\angle$O = $\angle$O (Same angle) and $\angle$A = $\angle$B (each 90°)

Therefore, $\triangle AOF \sim \triangle BOD$ (AA similarity)

So, $\frac{\text{OA}}{\text{OB}} = \frac{\text{FA}}{\text{DB}}$

Also, in $\triangle$FAC and $\triangle$EBC, $\angle$A = $\angle$B (Each 90°)

and $\angle$FCA = $\angle$ECB (Vertically opposite angles).

Therefore, $\triangle FAC \sim \triangle EBC$ (AA similarity).

So, $\frac{\text{FA}}{\text{EB}} = \frac{\text{AC}}{\text{BC}}$

But EB = DB (B is mid-point of DE)

So, $\frac{\text{FA}}{\text{DB}} = \frac{\text{AC}}{\text{BC}}$ (2)

Therefore, from (1) and (2), we have:

$\frac{\text{AC}}{\text{BC}} = \frac{\text{OA}}{\text{OB}}$

i.e. $\frac{OC-OA}{OB-OC} = \frac{OA}{OB}$
or OB . OC – OA . OB = OA . OB – OA . OC
or OB . OC + OA . OC = 2 OA . OB
or (OB + OA). OC = 2 OA . OB
or $\frac{1}{OA} + \frac{1}{OB} = \frac{2}{OC}$ [Dividing both the sides by OA . OB . OC]

33. Here roots are equal,
$\therefore D = B^2 - 4AC = 0$
Here, $A = 1 + m^2, B = 2mc, C = \left(c^2 - a^2\right)$
$\therefore (2mc)^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right) = 0$
or, $4m^2c^2 - 4\left(1 + m^2\right)\left(c^2 - a^2\right) = 0$
or, $m^2c^2 - \left(c^2 - a^2 + m^2c^2 - m^2n^2\right) = 0$
or, $m^2c^2 - c^2 + a^2 - m^2c^2 + m^2a^2 = 0$
or, $-c^2 + a^2 + m^2a^2 = 0$
or, $c^2 = a^2\left(1 + m^2\right)$
Hence Proved.

OR

Let P be the initial production (2 yr ago) and the increase in production in every year be x%.
Then, production at the end of the first year.
$P + \frac{Px}{100} = P(1 + \frac{x}{100})$
Production at the end of the second year = $P(1 + \frac{x}{100}) + \frac{x}{100}P[1 + \frac{x}{100}]$
$= P(1 + \frac{x}{100})(1 + \frac{x}{100})$
$= P(1 + \frac{x}{100})^2$
Since, production doubles in the last two years,
$\therefore P(1 + \frac{x}{100})^2 = 2P$
$\Rightarrow (1 + \frac{x}{100})^2 = 2$
$\Rightarrow (1 + \frac{x}{100}) = \sqrt{2}$
$\Rightarrow \frac{x}{100} = \sqrt{2} - 1 = 1.4142 - 1 = 0.4142$
$\Rightarrow x = 0.4142 \times 100$
$\Rightarrow x = 41.42\%$

34. Let the radii of the two circlular plots be r_1 and r_2, respectively.
Then, $r_1 + r_2 = 14$ [$\because$ Distance between the centres of two circlular plots = 14 cm,given].... (i)
Also, Sum of Areas of the plots = 130π
$\therefore \pi r_1^2 + \pi r_2^2 = 130\pi \Rightarrow r_1^2 + r_2^2 = 130$ ...(ii)
Now, from equation (i) and equation (ii),
$\Rightarrow (14 - r_2)^2 + r_2^2 = 130$
$\Rightarrow 196 - 2r_2 + 2r_2^2 = 130$
$\Rightarrow 66 - 2r_2 + 2r_2^2 = 0$
Solving the quadratic equation we get,
$r_2 = 3$ or $r_2 = 11$,
but from figure it is clear that, $r_1 > r_2$
$\therefore r_1 = 11$ cm and $r_2 = 3$ cm
The value depicted by the farmers are of cooperative nature and mutual understanding.

OR

We have to find upto three places of decimal the radius of the circle whose area is the sum of the areas of two triangles whose sides are 35, 53, 66 and 33, 56, 65 measured in centimetres.

For the first triangle, we have a = 35, b = 53 and c = 66.

$\therefore \quad s=\frac{a+b+c}{2}=\frac{35+53+66}{2}=77\text{cm}$

Let Δ_1 be the area of the first triangle. Then,

$\Delta_1=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_1=\sqrt{77(77-35)(77-53)(77-66)}=\sqrt{77\times 42\times 24\times 11}$

$\Rightarrow \quad \Delta_1=\sqrt{7\times 11\times 7\times 6\times 6\times 4\times 11}=$

$\sqrt{7^2\times 11^2\times 6^2\times 2^2}=7\times 11\times 6\times 2=924\text{cm}^2$ ...(i)

For the second triangle, we have a = 33,b = 56,c = 65

$\therefore \quad s=\frac{a+b+c}{2}=\frac{33+56+65}{2}=77\text{cm}$

Let Δ_2 be the area of the second triangle. Then,

$\Delta_2=\sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad \Delta_2=\sqrt{77(77-33)(77-56)(77-65)}$

$\Rightarrow \quad \Delta_2=\sqrt{77\times 44\times 21\times 12}=$

$\sqrt{7\times 11\times 4\times 11\times 3\times 7\times 3\times 4}=\sqrt{7^2\times 11^2\times 4^2\times 3^2}$

$\Rightarrow \quad \Delta_2=7\times 11\times 4\times 3=924\text{cm}^2$

Let r be the radius of the circle. Then,

Area of the circle = Sum of the areas of two triangles

$\Rightarrow \quad \pi r^2=\Delta_1+\Delta_2$

$\Rightarrow \quad \pi r^2=924+924$

$\Rightarrow \quad \frac{22}{7}\times r^2=1848$

$\Rightarrow \quad r^2=1848\times\frac{7}{22}=3\times 4\times 7\times 7\Rightarrow \ r=\sqrt{3\times 2^2\times 7^2}=2\times 7\times\sqrt{3}=14\sqrt{3}\text{cm}$

35. class 10000 - 15000 has the maximum frequency,

so it is the modal class.

$\therefore$ l = 10000, h = 5000, f = 41, f_1 = 26 and f_2 = 16

Mode = l + $\frac{f-f_1}{2f-f_1-f_2}\times$ h

= 10000 + $\frac{41-26}{2(41)-26-16}\times$ 5000

= 10000 + $\frac{15}{40}\times$ 5000

= 10000 + 1875

= 11875

Section E

36. **Read the text carefully and answer the questions:**

The students of a school decided to beautify the school on an annual day by fixing colourful flags on the straight passage of the school. They have 27 flags to be fixed at intervals of every 2 metre. The flags are stored at the position of the middlemost flag. Ruchi was given the responsibility of placing the flags. Ruchi kept her books where the flags were stored. She could carry only one flag at a time.

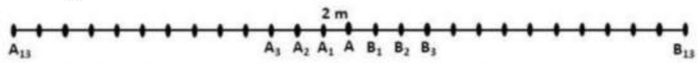
(i) Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$

$\Rightarrow S_6 = 3[8 + 20]$

$\Rightarrow S_6 = 84$

(ii) 

Let A be the position of the middle-most flag.

Now, there are 13 flags (A_1, A_2 ... A_{12}) to the left of A and 13 flags (B_1, B_2, B_3 ... B_{13}) to the right of A.

Distance covered in fixing flag to A_1 = 2 + 2 = 4 m

Distance covered in fixing flag to A_2 = 4 + 4 = 8 m

Distance covered in fixing flag to A_3 = 6 + 6 = 12 m

...

Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m

This forms an A.P. with,

First term, a = 4

Common difference, d = 4

and n = 13

(iii) $\therefore$ Distance covered in fixing 13 flags to the left of A = S_{13}

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$

$= \frac{13}{2} \times [8 + 48]$

$= \frac{13}{2} \times 56$

= 364

Similarly, distance covered in fixing 13 flags to the right of A = 364

Total distance covered by Ruchi in completing the task

= 364 + 364 = 728 m

OR

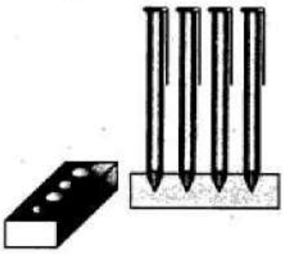
Maximum distance travelled by Ruchi in carrying a flag

= Distance from A_{13} to A or B_{13} to A = 26 m

37. **Read the text carefully and answer the questions:**

A carpenter in the small town of Bareilly used to make and sell different kinds of wood items like a rectangular box, cylindrical pen stand, and cuboidal pen stand. One day a student came to his shop and asked him to make a pen stand with the dimensions as follows:

A pen stand should be in the shape of a cuboid with four conical depressions to hold pens. The dimensions of the cuboid should be 15 cm by 10 cm by 3.5 cm. The radius of each of the depressions is 0.5 cm and the depth is 1.4 cm.

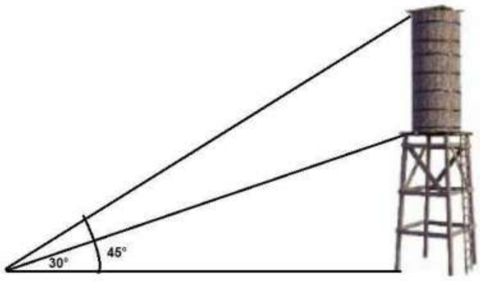


(i) Volume of the cuboid
$= 15 \times 10 \times 3.5 = 525 \text{cm}^3$
(ii) Volume of a conical depression
$= \frac{1}{3}\pi(0.5)^2(1.4)$
$= \frac{1}{3} \times \frac{22}{7} \times 0.25 \times \frac{14}{10} = \frac{11}{30}\text{cm}^3$
$\therefore$ Volume of four conical depressions
$= 4 \times \frac{11}{30}\text{cm}^3 = \frac{22}{15}\text{cm}^3 = 1.47\text{cm}^3$
(iii) $\therefore$ Volume of the wood in the entire stand = volume of cuboid - volume of 4 conical depressions
$= 525 - 1.47 = 523.53\text{cm}^3$

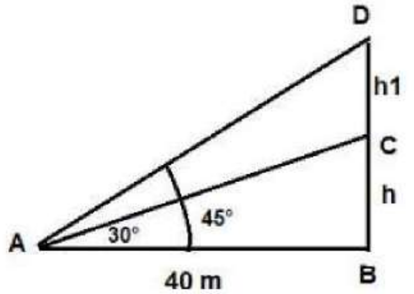
OR

Cost of wood per cm^3 = ₹10
Total cost of making pen stand = 10 × 523.53 = ₹5235.3

38. **Read the text carefully and answer the questions:**
In a society, there are many multistory buildings. The RWA of the society wants to install a tower and a water tank so that all the households can get water without using water pumps. For this they have measured the height of the tallest building in the society and now they want to install a tower that will be taller than that so that the level of water must be higher than the tallest building in their society. Here is one solution they have found and now they want to check if it will work or not.
From a point on the ground 40 m away from the foot of a tower, the angle of elevation of the top of the tower is 300. the angle of elevation of the top of the water tank is 45^{O}.



(i)

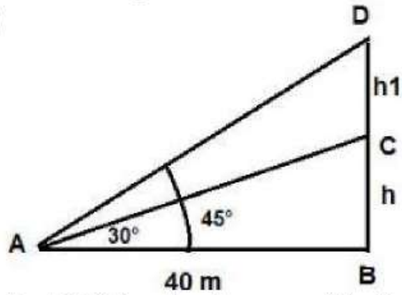


Let BC be the tower of height h and CD be the water tank of height h_1
In ΔABD, we have
$\tan 45^\circ = \frac{BD}{AB}$
$\Rightarrow 1 = \frac{h+h_1}{40}$
$\Rightarrow h + h_1 = 40$ ...(1)
In ΔABC, we have
$\tan 30^\circ = \frac{BC}{AB}$
$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.1$ m

Thus height of the tower is 23.1 m.

(ii)



Let BC be the tower of height h and CD be the water tank of height h_1

In ΔABD, we have

$\tan 45^\circ = \frac{BD}{AB}$

$\Rightarrow 1 = \frac{h+h_1}{40}$

$\Rightarrow h + h_1 = 40$ ...(1)

In ΔABC, we have

$\tan 30^\circ = \frac{BC}{AB}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40}$

$\Rightarrow h = \frac{40}{\sqrt{3}} = \frac{40\sqrt{3}}{3} = 23.1$ m

Thus height of the tower is 23.1 m.

Substituting the value of h in (1), we have

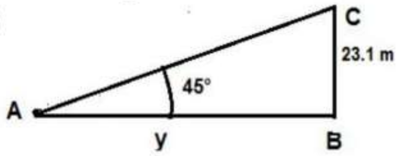
$23.1 + h_1 = 40$

$\Rightarrow h_1 = 40 - 23.1$

$= 6.9$ m

Thus height of the tank is 6.9 m.

(iii)



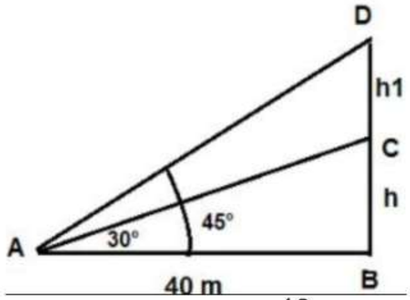
In the $\triangle ABC$ if $\angle CAB = 45^\circ$ then

$\cot 45^\circ = \frac{y}{23.1} = 1$

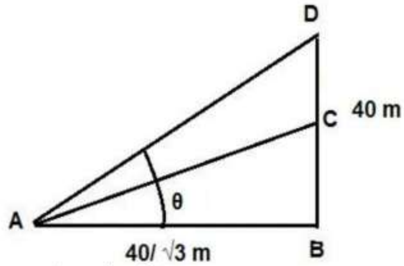
y = 23.1 m

Thus the angle of elevation will be 45^o at 23.1 m.

OR



Given that $AB = \frac{40}{\sqrt{3}}$



In the $\triangle ABD$

$\cot \theta = \frac{AB}{BD} = \frac{\frac{40}{\sqrt{3}}}{40}$

$\cot \theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \theta = 60^{\circ}$

Hence the angle of elevation would be 60^{0}.